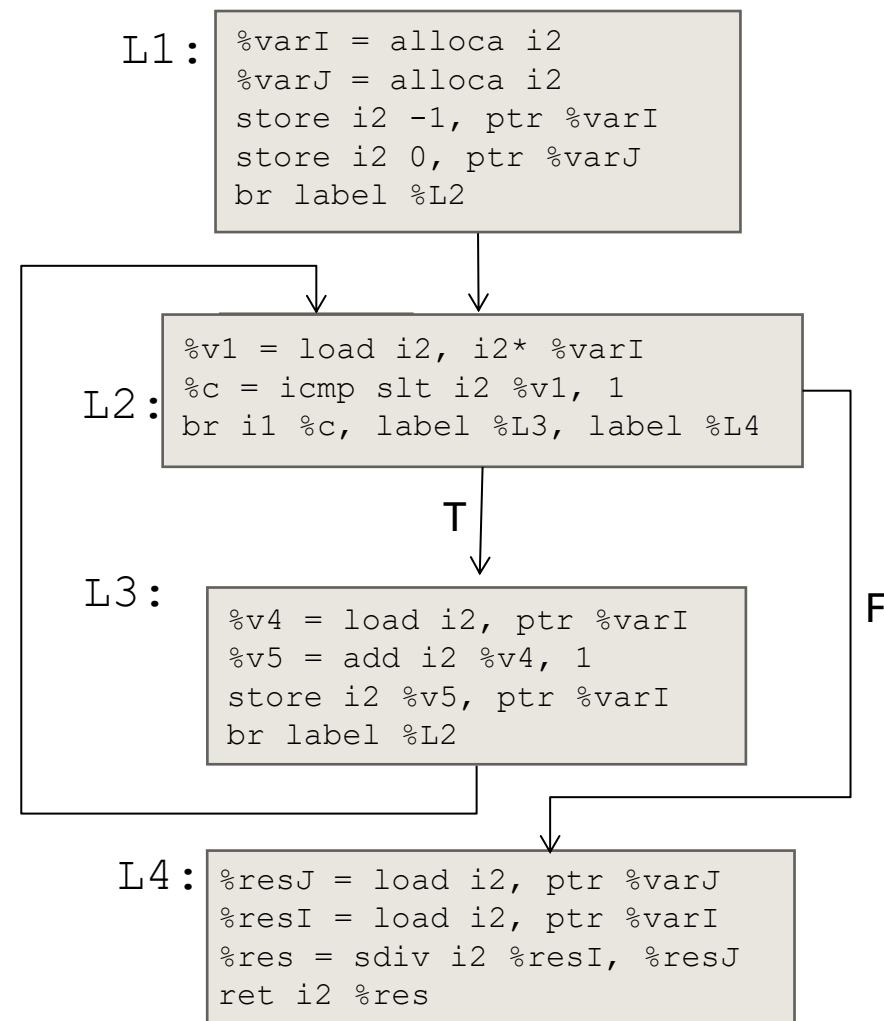


EXERCISE

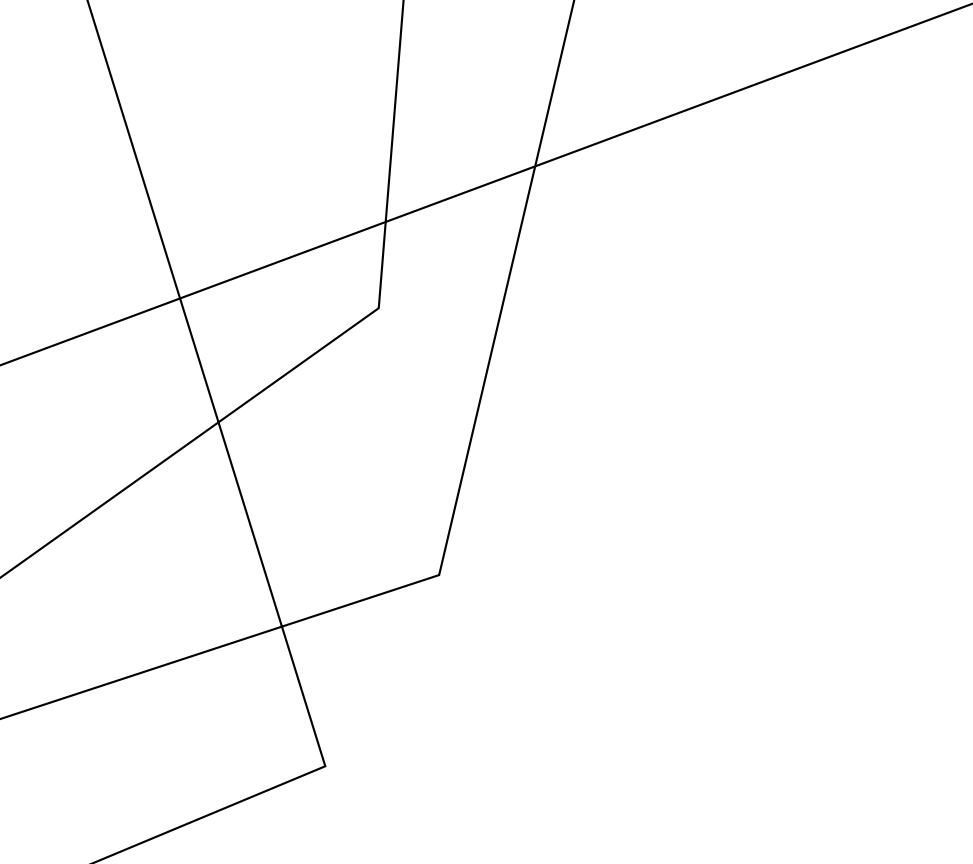
Consider the CFG to the right, roughly equivalent to the C code below.

Indicate the value-set computed for `*varJ` at the entrance to L4 using the flow-sensitive saturation algorithm described in the last lecture

```
int2 i = -1;
int2 j = 0;
while (i < 1){
    i++;
    j = 1;
}
return i/j;
```



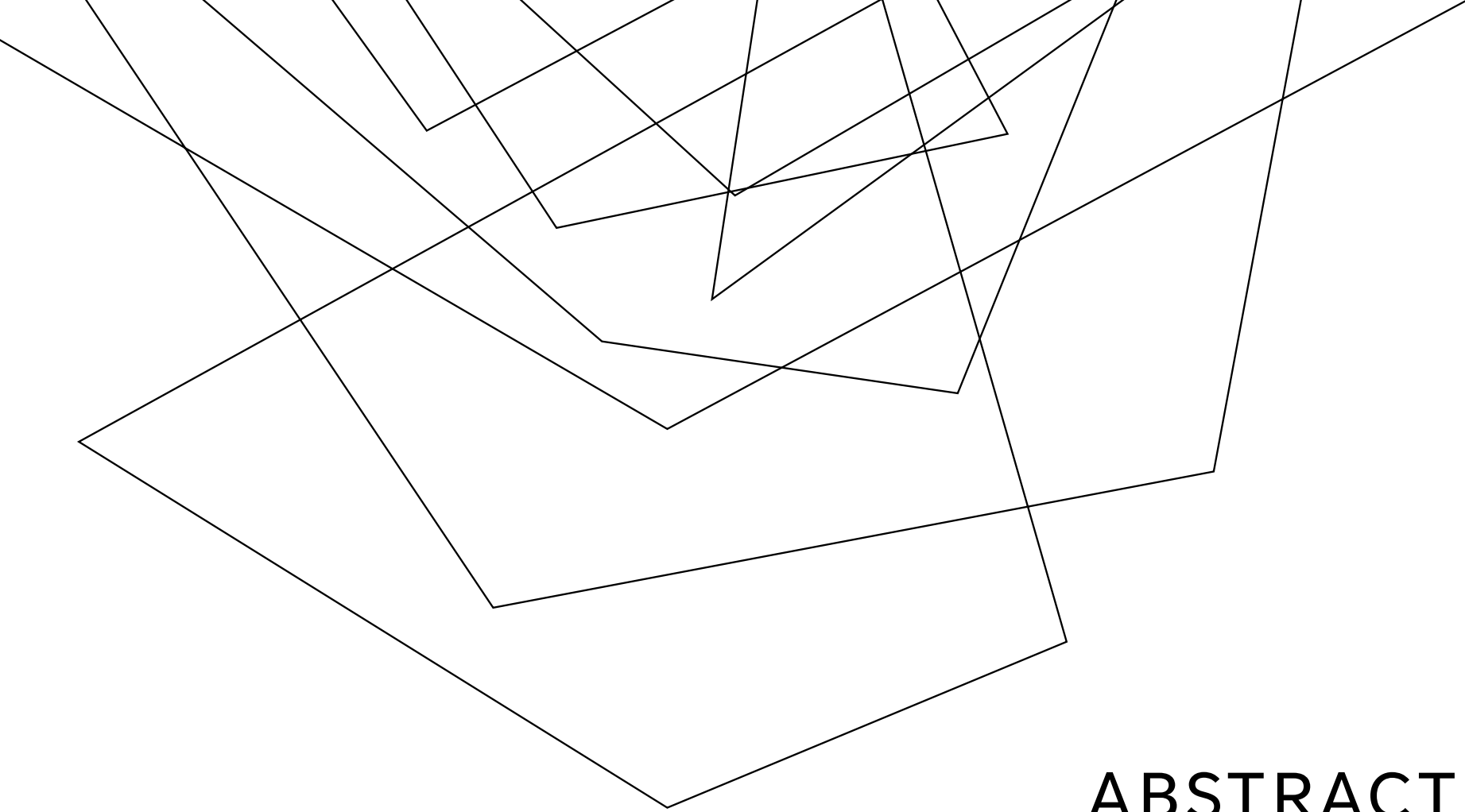
EXERCISE: SOLUTION



ADMINISTRIVIA AND ANNOUNCEMENTS

Test 1 is Friday!

Review Session tomorrow, 7 – 9, LEEP2 2420

Abstract geometric lines in the top-left corner of the slide, consisting of several thin black lines forming overlapping, irregular polygons and triangles.

ABSTRACT INTERPRETATION

EECS 677: Software Security Evaluation

Drew Davidson

LAST TIME: HANDLING LOOPS

REVIEW: STATIC ANALYSIS

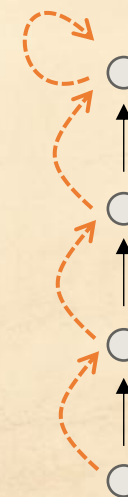
EXTENDING OUR BASIC DATAFLOW TO LOOPS

Problems

- Circular data dependence
Introduce a TBD Value - ??? or $\perp(\text{ツ})\text{/}$
- No obvious end-point (will the analysis halt?)
Ran analysis until fact-set saturation

Guaranteeing Saturation

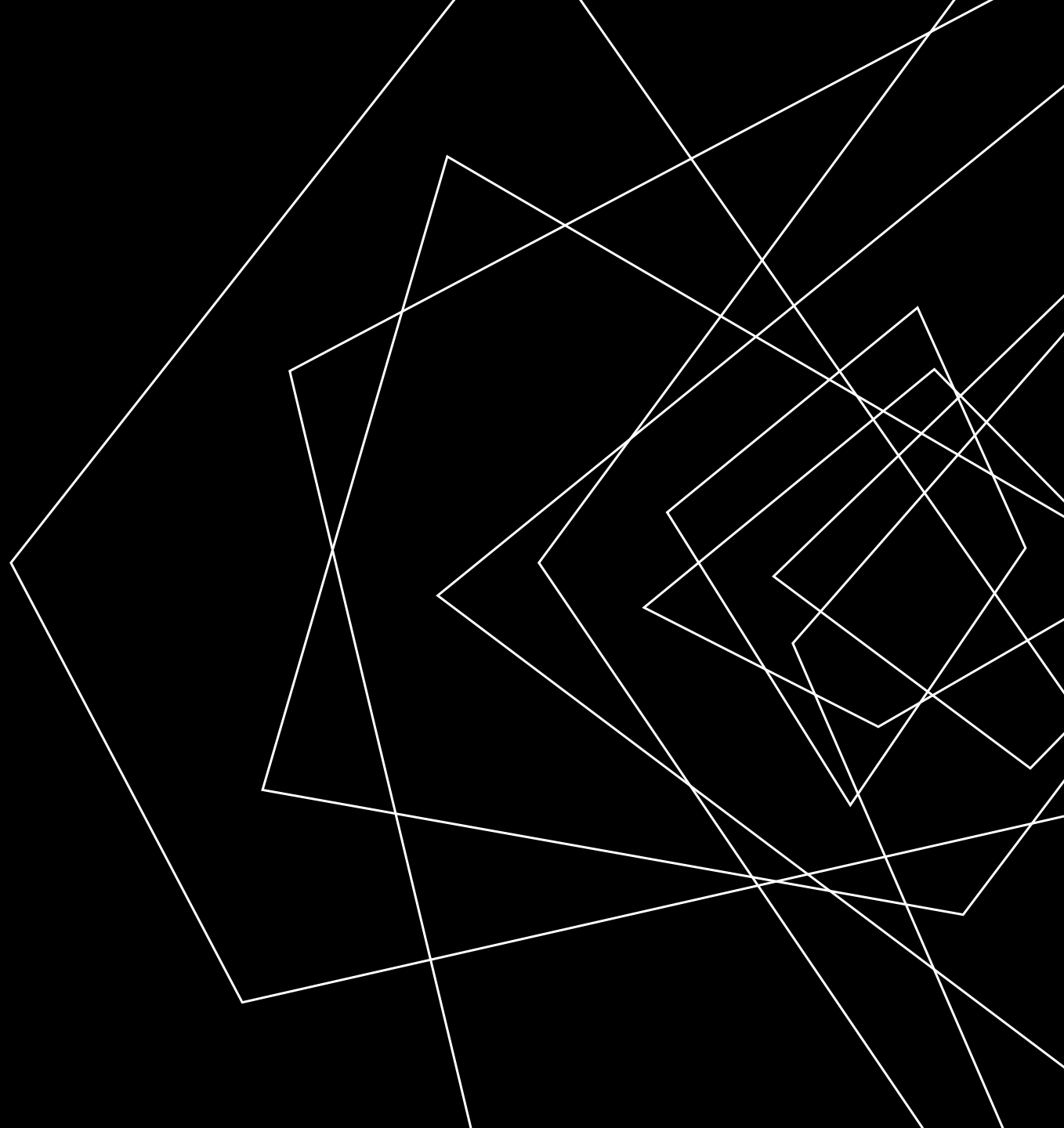
- Applying a transform is constant-time
- The fact sets are finite
- The fact sets never shrink



*Perform an operation until it
stops making progress*

LECTURE OUTLINE

- Formalizing Saturation
- Exploring Larger Programs
- Abstract Domains



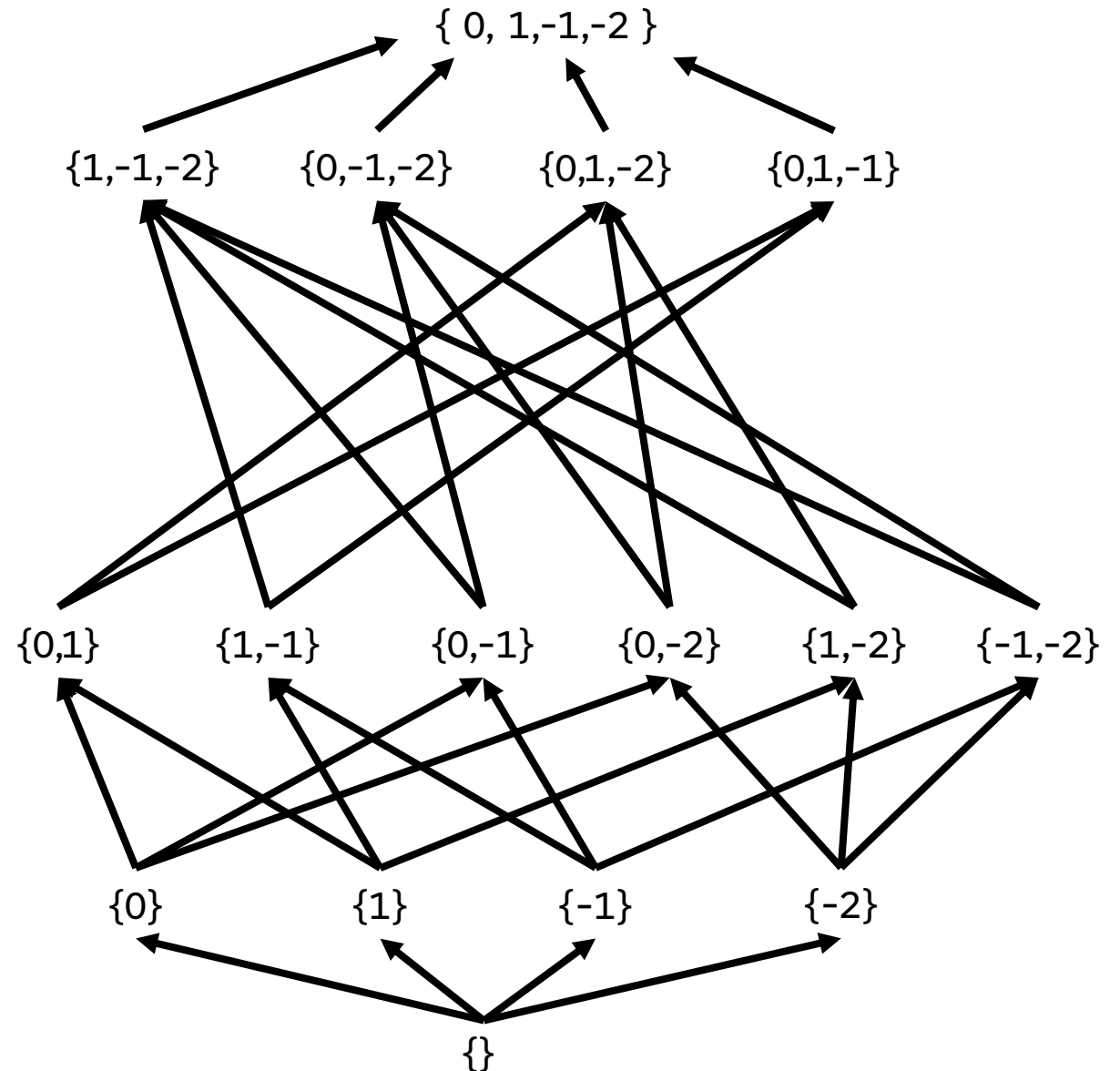
FACT SET MONOTONICITY

DATAFLOW FRAMEWORKS

WHAT DOES IT MEAN THAT A FACT SET
“NEVER SHRINKS”?

- Implicitly, partial ordering by set inclusion...
- Merge operation (here, union) never moves down the partial ordering

Connecting to formal properties
provides the guarantees we crave



DOMAIN NEEDS

DATAFLOW FRAMEWORKS

SOME BASIC DEFINITIONS

A **partially-ordered set** (poset) is a set S and a partial ordering $\subseteq$, such that the ordering $\subseteq$ is:

- Reflexive
- Anti-symmetric
- Transitive

A **lattice** is a poset in which each pair of elements has

- A least upper bound (the *join*)

for x and y , the join z is defined such that:

- $x \subseteq z$ and
- $y \subseteq z$ and

- for all w such that $x \subseteq w$ and $y \subseteq w$, $w \supseteq z$

*No upper bound
lower than z*

z is actually an upper bound

- A greatest lower bound (the *meet*)

basically the same deal, but reversed

A **complete lattice** is a lattice in which all subsets have a meet and join

Example 1: S : English words, $\subseteq$ substring

Poset: ✓ Lattice: ✗

Example 2: S : English words, $\subseteq$ shorter or equal in length

Poset: ✗ Lattice: ✗

Example 3: S : integers, $\subseteq$ as lte

Poset: ✓ Lattice: ✓

Example 4: S : integers, $\subseteq$ as lt

Poset: ✗ Lattice: ✗

Example 5: S : set of all sets of letters, $\subseteq$ is subset

Poset: ✓ Lattice: ✓

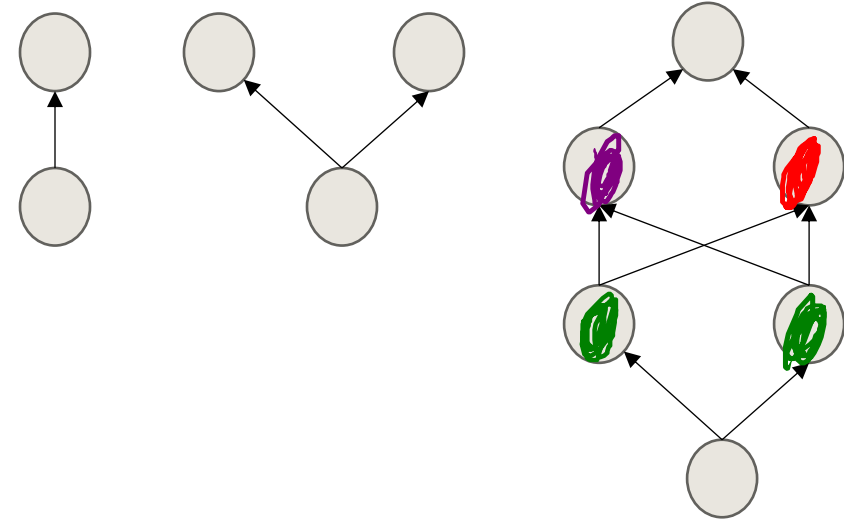
FACT DATATYPE NEEDS

DATAFLOW FRAMEWORKS

FORMAL(ISH) REQUIREMENTS

- A fact datatype (ideally of unbounded size)
- An ordering that indicates progress
- Unique solution
- A guarantee of a finite number of steps to hit the maximum value
- An update step that never loses progress

POSET



TRY-OUTS

- poset
- Lattice
- Complete lattice

FACT DATATYPE NEEDS

DATAFLOW FRAMEWORKS

FORMAL(ISH) REQUIREMENTS

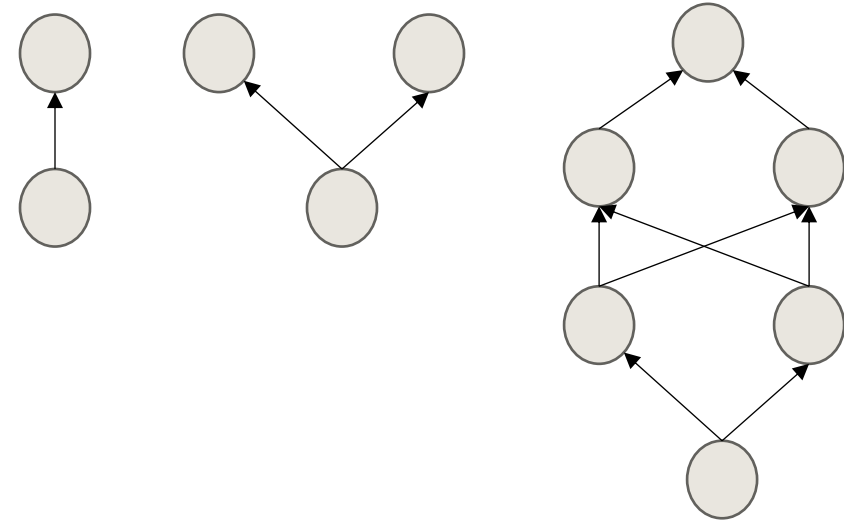
- A fact datatype (ideally of unbounded size)
- An ordering that indicates progress
- Unique solution
- A guarantee of a finite number of steps to hit the maximum value
- An update step that never loses progress

TRY-OUTS

- poset
- Lattice
- Complete lattice

LATTICE

Poset with a least upper bound and a greatest lower bound



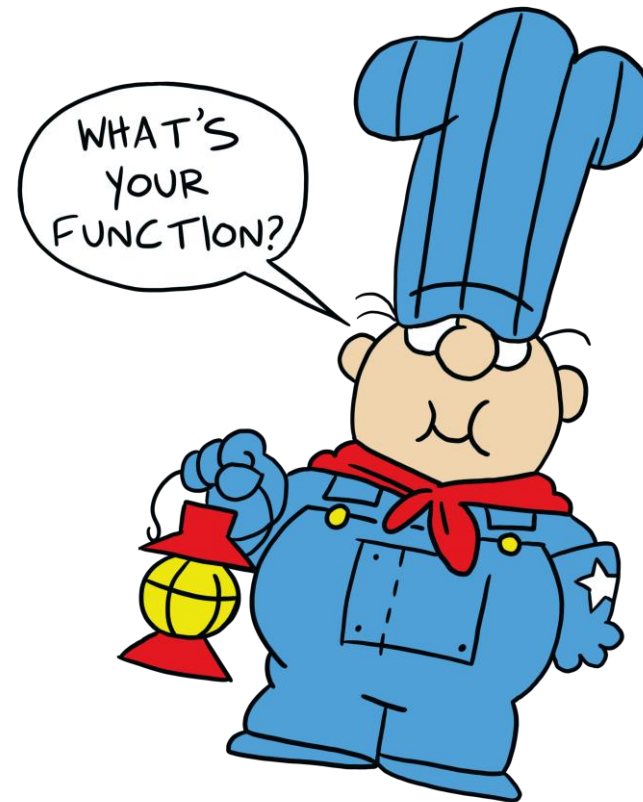
FUNCTION NEEDS

DATAFLOW FRAMEWORKS

SOME BASIC DEFINITIONS

A function f is a **monotonic function** if
 $x \subseteq y$ implies $f(x) \subseteq f(y)$

An element z is a **fixpoint** of f iff $z = f(z)$



FUNCTION NEEDS

DATAFLOW FRAMEWORKS

SOME BASIC DEFINITIONS

A function f is a **monotonic function** if
 $x \subseteq y$ implies $f(x) \subseteq f(y)$

An element z is a **fixpoint** of f iff $z = f(z)$

Example

$$f(x) = x \cup \{ b \}$$

$$f(\{b\}) = \{ b \} \longleftarrow \{b\} \text{ is a fixpoint of } f$$

$$f(\{a\}) = \{ a, b \} \longleftarrow \{a\} \text{ is not a fixpoint of } f$$

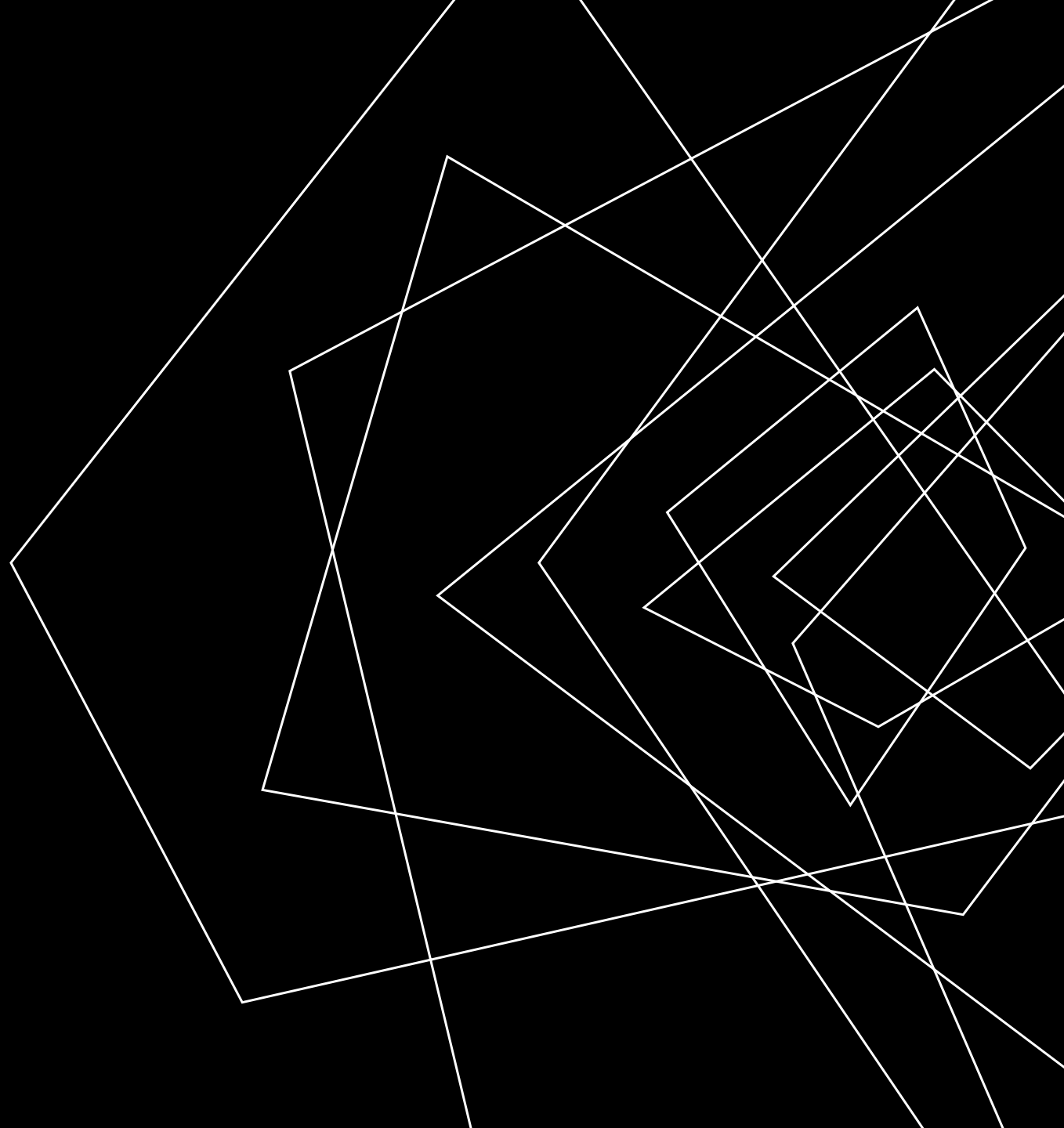
$$f(\{a, b\}) = \{ a, b \} \longleftarrow \{a, b\} \text{ is a fixpoint of } f$$

$$f(f(\{a\})) \text{ is a fixpoint of } f$$

$$f(f(\text{any set})) \text{ is a fixpoint of } f$$

LECTURE OUTLINE

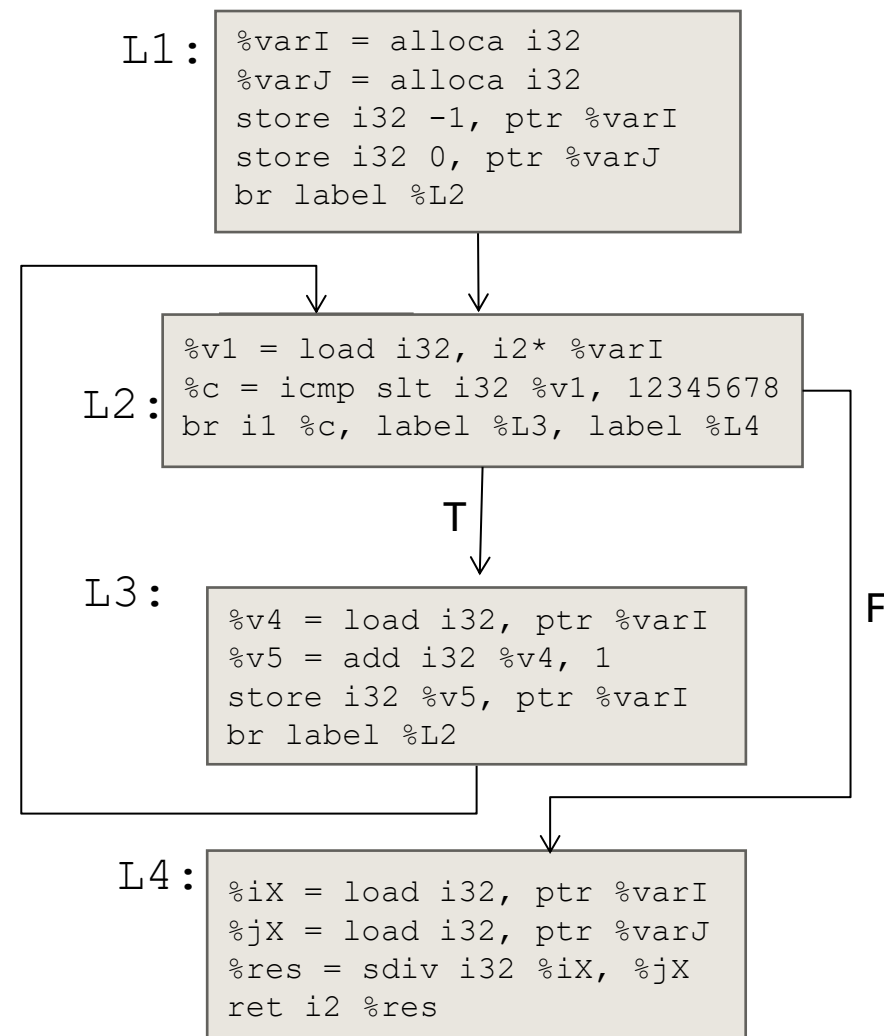
- Formalizing Saturation
- Exploring Larger Programs
- Abstract Domains



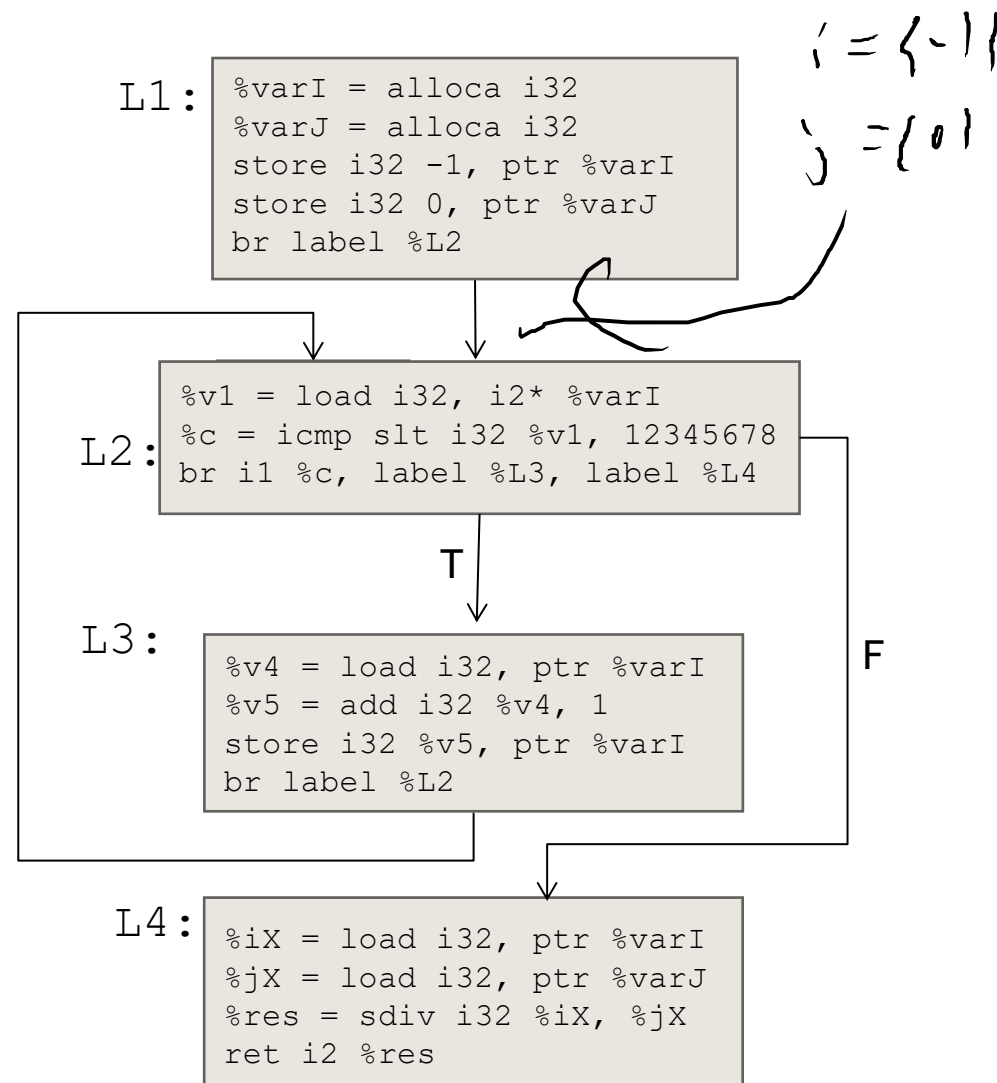
EXAMPLE

Consider the CFG to the right, roughly equivalent to the C code below.
Indicate the value-set computed for `*varJ` at the end of L6 using the flow-sensitive saturation algorithm described in the last lecture

```
int32 i = -1;
int32 j = 0;
while (i > 12345678){
    i++;
}
return 1/j;
```



EXAMPLE

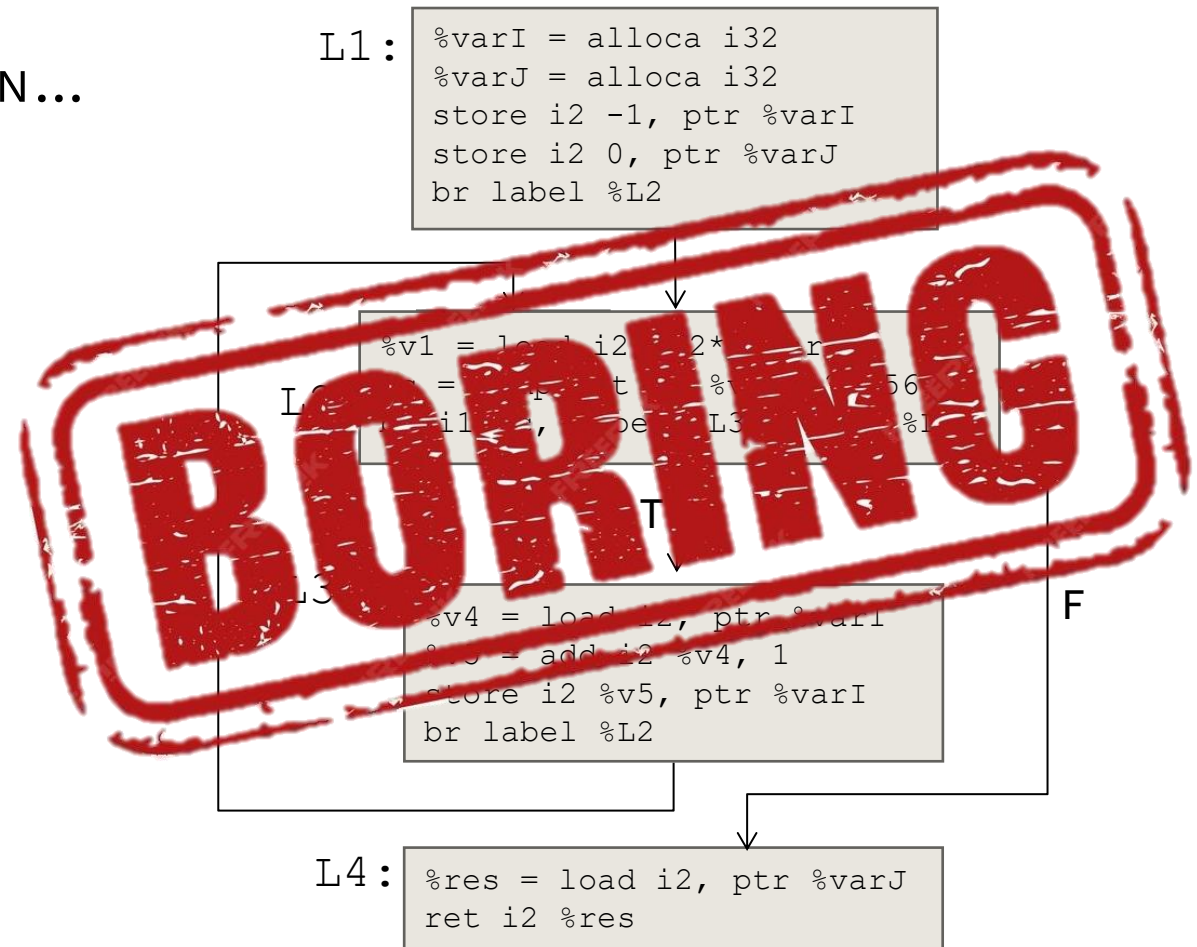


TMI – TOO MUCH INFORMATION!

ABSTRACT INTERPRETATION

EVENTUALLY WE'LL REACH SATURATION...

By the time we're all old and withered!

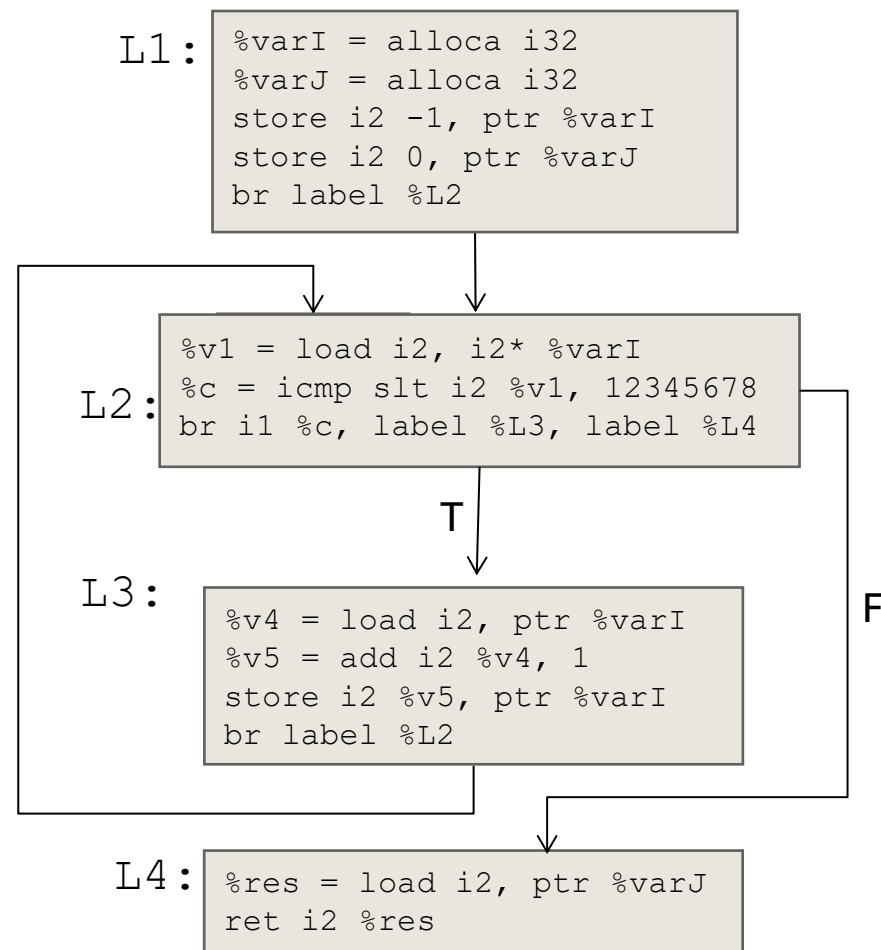


TMI – TOO MUCH INFORMATION!

ABSTRACT INTERPRETATION

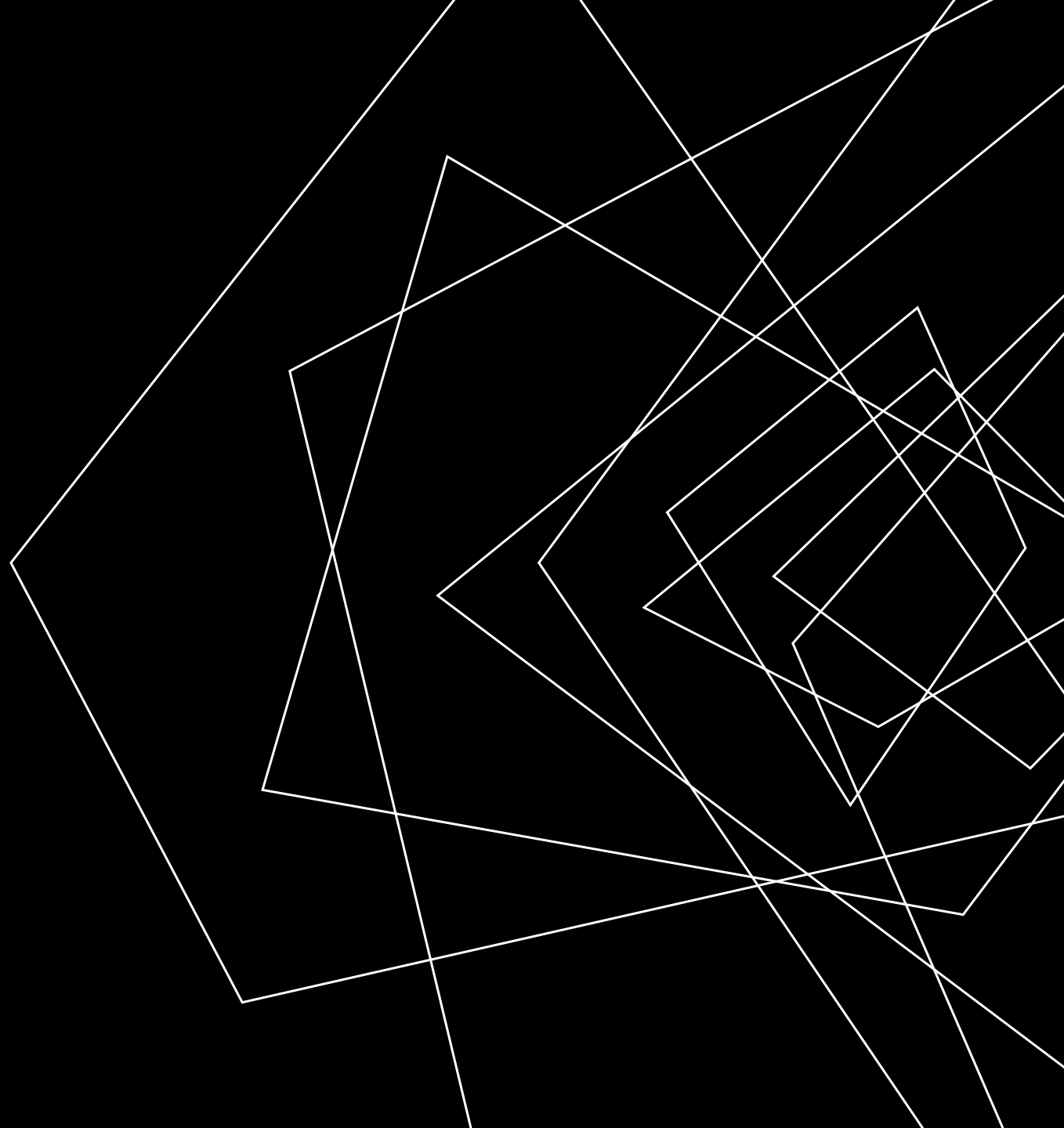
EVENTUALLY WE'LL REACH SATURATION...

By the time we're all old and withered!



LECTURE OUTLINE

- Formalizing Saturation
- Exploring Larger Programs
- Abstract Domains



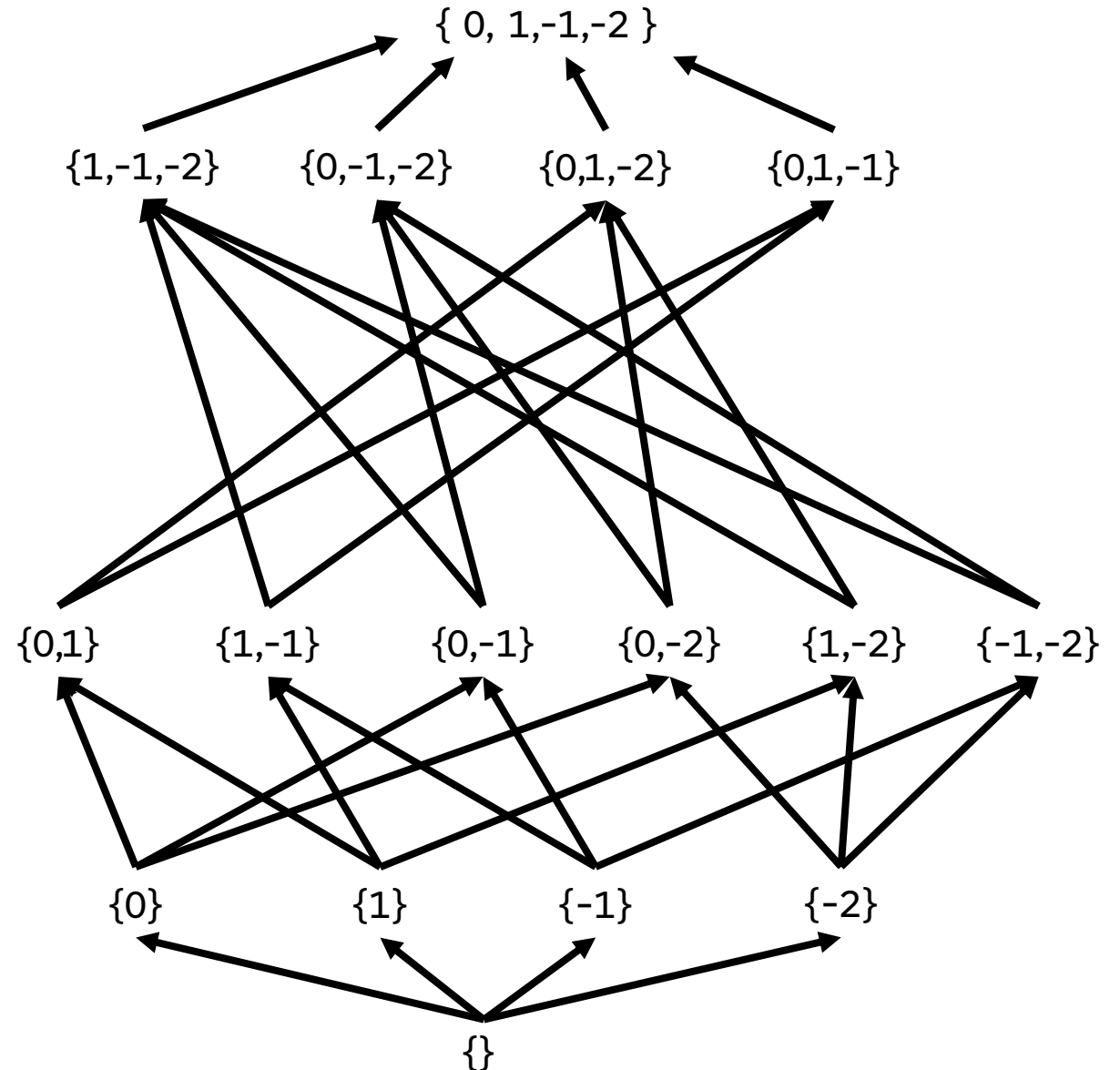
USING THE FORMALISMS

DATAFLOW FRAMEWORKS

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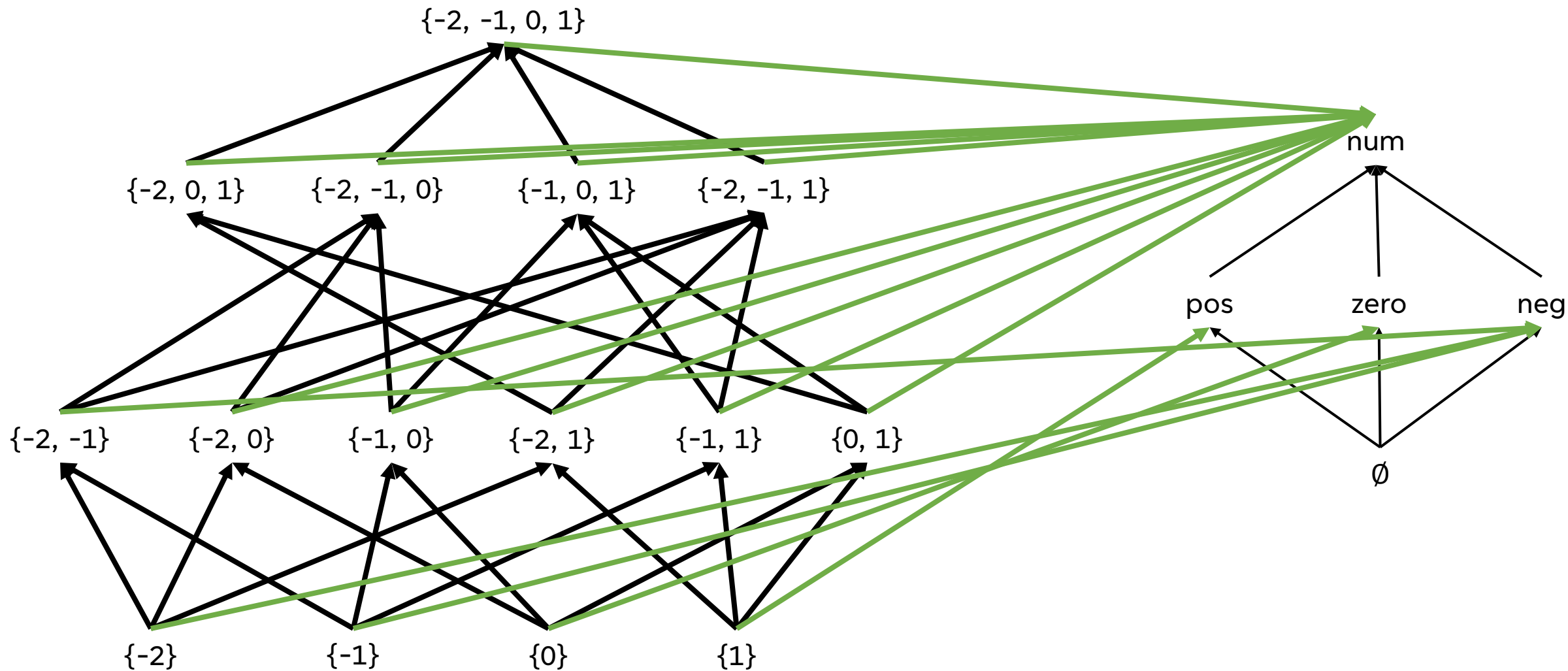
Connecting to formal properties
provides the guarantees we crave



A VERY APPROXIMATE LATTICE

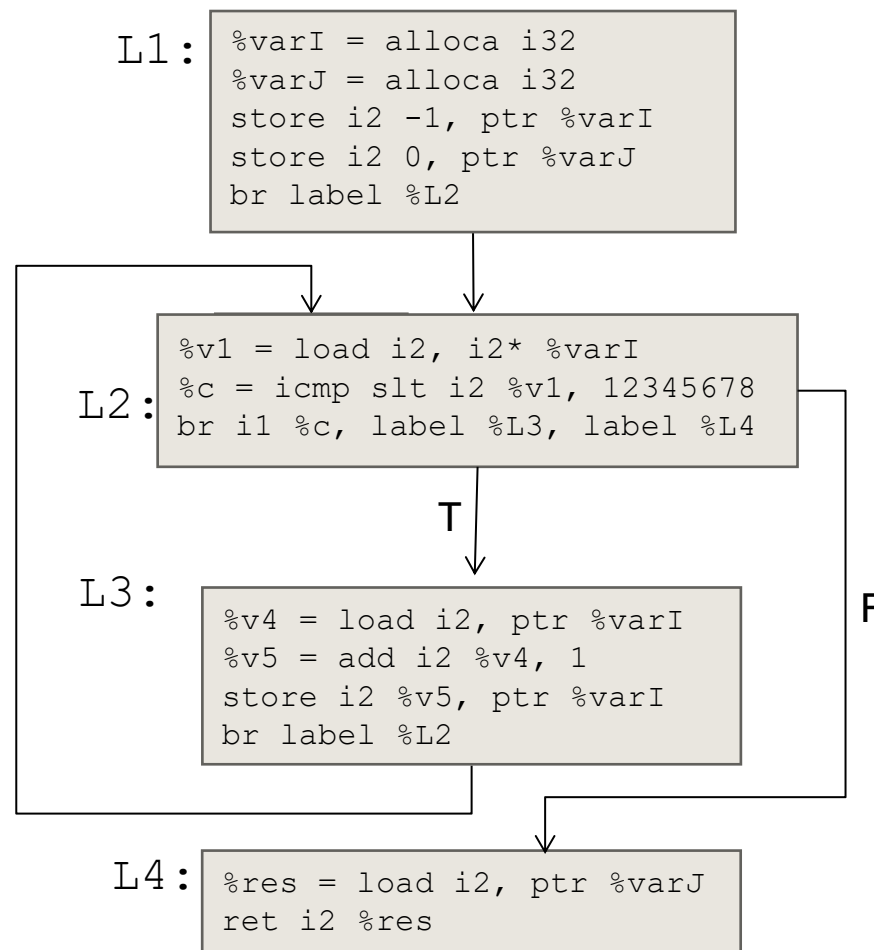
ABSTRACT INTERPRETATION

ABSTRACT DOMAIN OF SIGNS



ANALYSIS WITH ABSTRACT DOMAIN OF SIGNS

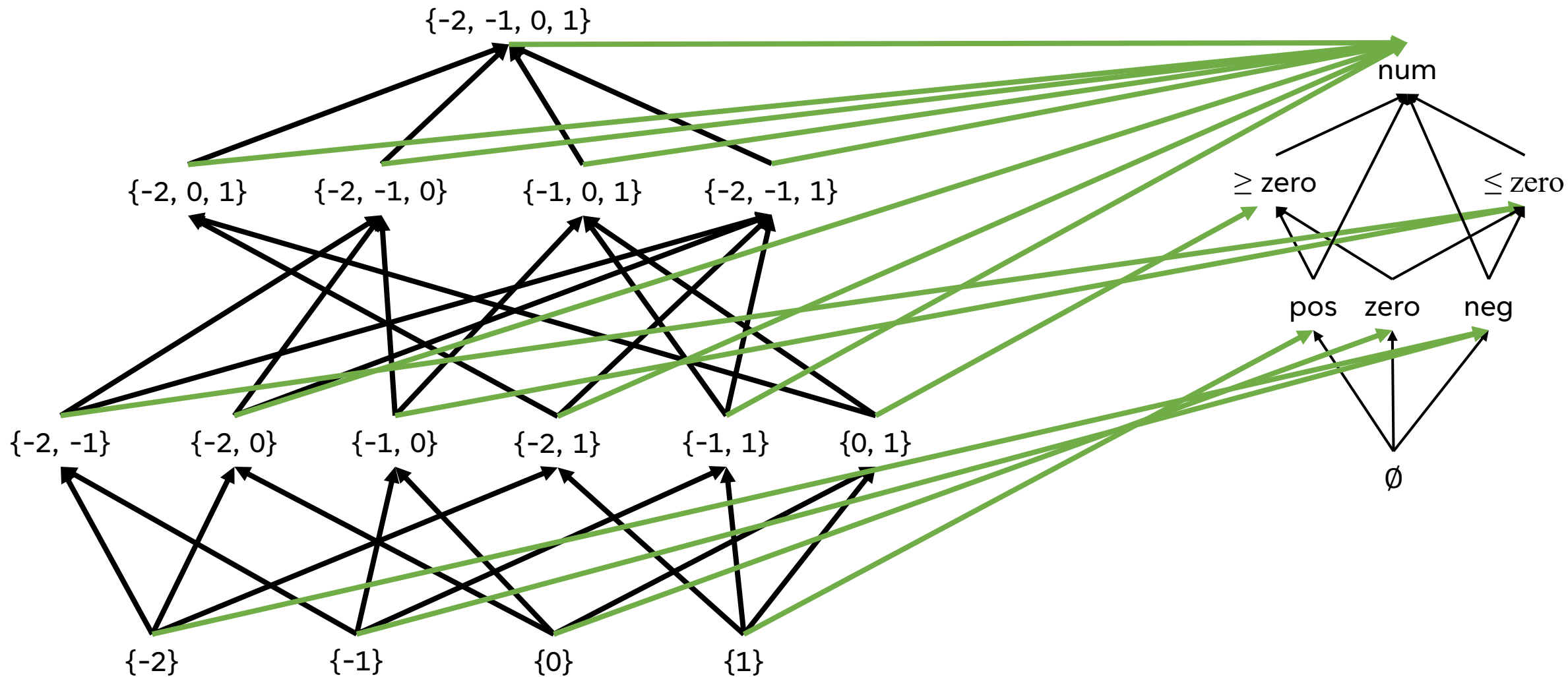
ABSTRACT INTERPRETATION



A LESS-APPROXIMATE LATTICE

ABSTRACT INTERPRETATION

ABSTRACT DOMAIN OF SIGNS



ABSTRACT DOMAINS IN PRACTICE

STATIC ANALYSIS

“SINGLETON INTEGER SETS”

- You know the number, or you don't

INTERVALS

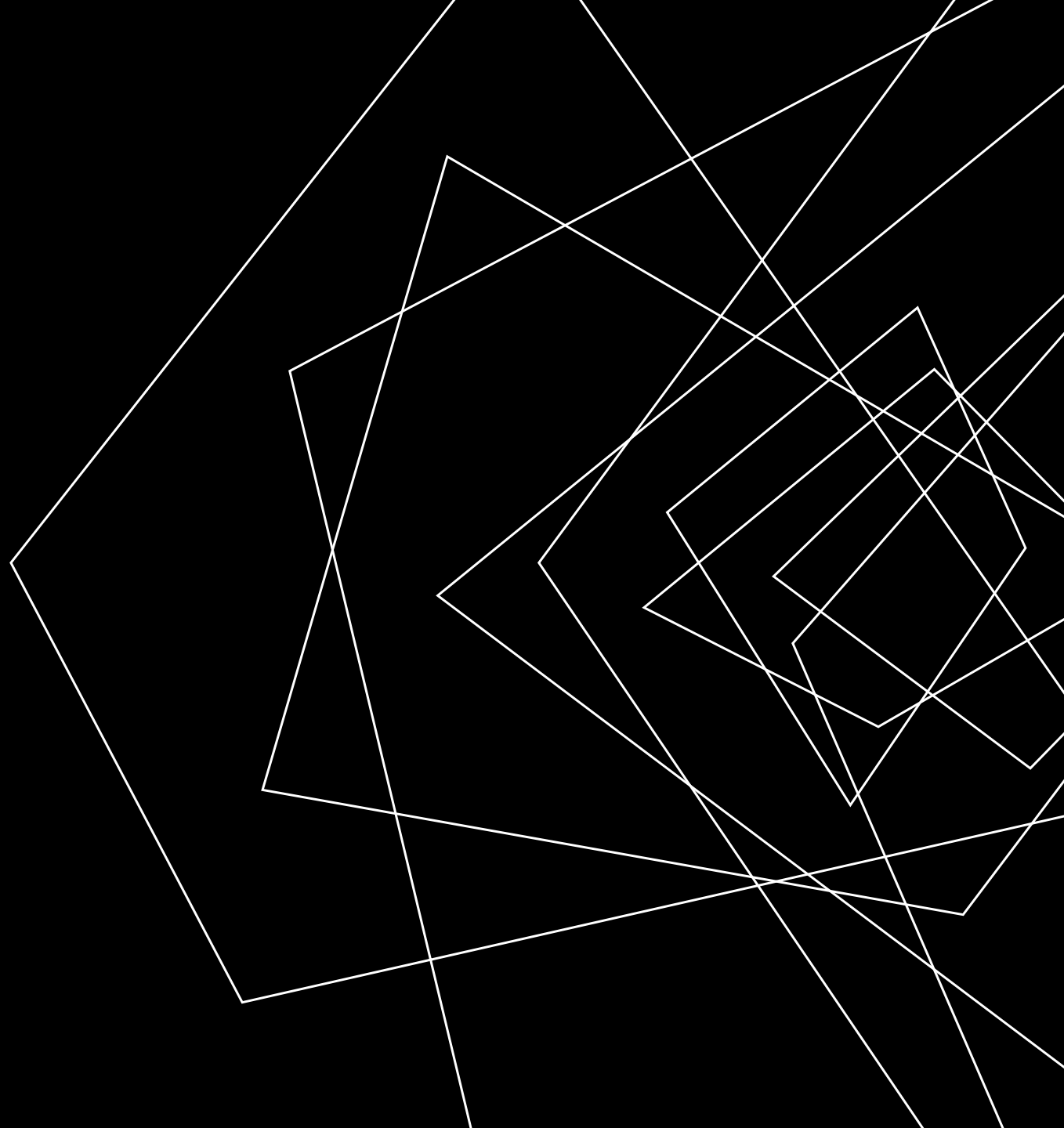
- You know a concrete range

PROPERTY EXISTENCE

- A property does or does not hold

LECTURE OUTLINE

- Enhancing Dataflow analysis
- Lattices
- Abstract Interpretation



SECTION SUMMARY

STATIC ANALYSIS

STATIC ANALYSIS GIVES US AN IMPORTANT GUARANTEE

- Completeness of bug finding /
Soundness of verification

Anything that isn't crystal clear to a static analysis tool probably isn't clear to your fellow programmers, either. The classic hacker disdain for "bondage and discipline languages" is short-sighted – the needs of large, long-lived, multi-programmer projects are just different than the quick work you do for yourself.

[- John Carmack's Static Code Analysis post](#)